Further Maths Revision Paper 3

This paper consists of 5 questions covering CP1, CP2, FP1 and FM1.
(AS Further Maths: Q1 and 3)

1

$$P = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$$

The matrix P represents a linear transformation, T, of the plane.

(a) Describe the invariant points of the transformation T.

(b) Describe the invariant lines of the transformation T.

2

The point P lies on the hyperbola with equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

The points S and S' are the foci of the hyperbola.
Show that $S'P - SP = 2a$

3

(a) Obtain the Cartesian equation of the straight line which passes through the point $A(-1, 2, 3)$ and which is normal to the plane $2x - 3y + 4z + 8 = 0$

(b) Calculate the coordinates of P the point of the intersection of this line with the plane.

(c) If the point $B(a, 2a, 3)$ lies on the plane, find the value of a and calculate the angle between AP and AB in degrees giving your answer to 1 decimal place.

4

A red ball is stationary on a rectangular billiard table $OABC$.
It is then struck by a white ball of equal mass and equal radius with velocity $u(-2\mathbf{i} + 11\mathbf{j})$ where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along OA and OC respectively.
After impact the red and white balls have velocities parallel to the vector $-3\mathbf{i}+4\mathbf{j}, 2\mathbf{i}+4\mathbf{j}$ respectively.

Show that the lines of centres on impact is parallel to $-3\mathbf{i} + 4\mathbf{j}$

5

Use Taylor's theorem to evaluate

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{(x - \frac{\pi}{2})}$$

You may use:

$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$